On the east side of the valley, perhaps 150 feet above the level of the road at Lie Station, I could distinctly trace this terrace by its hummocks of water-laid sand, and the farmhouses perched on its favourable points. A long series of hamlets on the road to Molde is placed upon it. As an object in physical geography, in its form, its uniform level on both sides of the vale, and its relation to the lakes at the summit-level, this terrace precisely resembles the lowest of the Glenroy terraces as it approaches Loch Laggan. It must, however, be more than twice the level above the sea" (p. 105). Chambers, of course, viewed it as an ancient sea-margin.

The same long terrace was also seen by my colleague, Mr. J. R. Dakyns, in 1872, and described (without reference to Chambers) in the *Geological Magazine*, 1877, p. 74. "If the terrace is on a level," he says, "with the watershed, and there is certainly no great difference between them, one is irresistibly led to think of the similar case of the parallel roads of Glenroy, and . . . of a gigantic Marjelen See dammed back by ice till it overflowed the summit of the pass at Molmen. It is significant that I could see no trace of terrace or water-mark on the Romsdal side of the pass. There is in the same district a second horizontal mark on the solid rock, several hundred feet higher than the 2000-feet one. This, too, seems to correspond with sand-terraces in the recesses of the high glens. . . . Here again it is striking that the water-mark should seem to correspond with the level of a watershed."

I myself saw Chambers's striking terrace in 1873. But I have nothing to add to the observations above quoted, and I make no claim whatever to have my name connected with them. But I may remark the fact that the little deltas or alluvial cones of the streams, where these cross the terraces, so conspicuously bear reference to the surface of the vanished sheet of water in which they were formed, as to remind one how greatly similar evidence was relied on by Darwin as demonstrating the *aqueous* origin of the roads of Glenroy. Mr. Hansen's discovery of parallel roads at the head of the Glommen and in Jemtland is very interesting, and I hope he will find time to study and *map* them in detail.

HUGH MILLER

51, Lauriston Place, Edinburgh, January 24

Meteorological Phenomena

ON January 4 last, while watching a very bright rainbow with a good secondary from Hoylake racecourse, I observed between the two bows a third, fainter than either, touching the primary at the base and extending upwards in such a way that probably, had it all been visible, it would have touched the secondary at the vertex. It was not all visible because of a break in the clouds. Its colours were in the same order as those of the primary, red outside. This third bow was only visible at one side; but a gentleman who observed it stated that he had seen it before, and symmetrical on both sides, though not extending to the vertex.

Another phenomenon I have observed here some time ago. A fall of hail lasting a few minutes occurred, the hailstones being exact cubes, of size about 7 mm. and of consistency like lumps of salt.

JOHN C. WILLIS

University College, Liverpool, February 1

M BARRÉ DE SAINT-VENANT

"WE have now to consider the earlier work of the greatest of living elasticians." Within a fortnight after these words were sent to the press, on January 6, M. de Saint-Venant died at Vendôme. The news of his death will have caused a deep feeling of regret among English mathematicians and physicists, to whom his researches are so well known that they have attained in their own field a classical value. We purpose in this notice to give some brief account of this foremost representative of latter-day French mathematical physicists.

Saint-Venant stood out for the younger mathematicians of the English school, as the link between the past and the present. Intimately related to the great period of French mathematical physics, he had continued to produce down to our own day, and we felt him to be as real a personality as Helmholtz or Thomson. A younger member of the school of Poisson, Navier, and Cauchy, he had yet lived to "edit" Clebsch. Deputy for Coriolis at the École des Ponts et Chaussées in 1837, Saint-Venant early received public recognition for his work from Poncelet in his lectures at the Sorbonne in 1840; within the next few years he corrected Cauchy's theory of torsion, and saw his correction accepted by the author of the "Exercices des mathématiques." More than forty years afterwards he "edits" what will long remain the standard treatise on elasticity—the annotated French translation of Clebsch. Thus his work is spread without a break across the middle fifty years of our century; he took up elasticity where Poisson had left it—a mathematical theory; he leaves it one of the most powerful branches of mathematics applied to physics and practical engineering; not a small amount of this transformation is due directly to his researches, or indirectly to his influence.

Turning to the personal character of the man, we find in him the essential characteristics of the scholar and the student, the truest modesty, the complete absence of self, the single-minded devotion to his study. Saint-Venant, whose researches on elasticity undoubtedly far surpass those of Navier and Clebsch, is yet content to appear as their *editor*. But what an editing it is! The original text is hidden, disappears, almost as completely as Peter the Lombard's "Sententia" in a mediæval commentary. It is Saint-Venant's notes, appendices, and corrections, which form the value of these works, which make the third edition of Navier's "Leçons" the standard treatise on the strength of materials, and the French translation of Clebsch the foremost work on mathematical elasticity. Nay, he even praises Clebsch for inventing a term in 1862, which he himself had first proposed in the privately distributed lithographed sheets of 1837! Ever ready with advice and assistance, perfectly free from jealousy, Saint-Venant was a typical scholar. We had occasion, scarcely six months ago, to apply to him for assistance with regard to some of his earlier work. Within a few days we received a packet containing twenty-three of his memoirs, all carefully corrected, and many annotated. He expressed a lively interest in the progress of the "History of the Mathematical Theories of Elasticity," lending the editor of that work several French lithographed courses which were otherwise inaccessible, and accompanying them by letters which amounted almost to a dissertation on the history of elasticity.

"Je desire, bien cher monsieur, que ces quelques renseignements et documents puissent servir à l'utile travail historique que vous avez entrepris, et dont j'apprendrai avec plaisir la publication ainsi que le nom de l'éditeur. J'en verrais même avec plaisir les épreuves."

Shortly before Christmas we received from Saint-Venant corrections for the first three sheets of Dr. Todhunter's ninth chapter, which is devoted to Saint-Venant's earlier work. On January 3 we sent him the remaining proofs of that chapter; a week afterwards we had to mourn the loss of one whose personal kindness had served to intensify the respect raised by his transcendent mathematical ability.

If we examine the leading characteristics of Saint-Venant's scientific work we find them marked by an essentially practical character. We find subtlety of analysis combined always with practical physical conceptions. The problems he attacks are those which are physically possible, or of which the solution is an immediate practical need. He smiles good-naturedly over Lamé's attempts to solve the terrible problem of an elastic solid in the form of a right-six-face, whose surface is subjected to any system of load. The solution would be a triumph of analysis, but its physical and practical value would in all probability be *nil*. He chooses instead a *real* beam, and he obtains a solution which, if it be but approximate, is at least an approximation to reality, and will serve all practical purposes. Saint-Venant never troubled himself with impossible distributions of load over impossible surfaces, but took the problems of mechanics as they occurred practically, and solved them for practical purposes. This tendency on his part was no doubt greatly due to his training as an engineer. He was Ingénieur-en-chef des Ponts et Chaussées; he had been Professeur de Génie rural à l'Institut agronomique; he had built lock-gates and improved the gutters of Paris; he was an authority on agricultural drainage, and had investigated the best form of the ploughshare; he designed a bridge for the Creuse, and planned a method, afterwards adopted, for drying up the vast marshes of the Sologne. Yet with all this he was a great master of analysis, and knew how to make his analysis fruitful in practice.

It is not our purpose here to give a bibliographical account[1] of Saint-Venant's works; we wish only to sketch the general scope of his researches, and shall confine ourselves to indicating the more important advances he has made in his own peculiar subject—that of elasticity. The first important work by Saint-Venant is the "Cours lithographée" of 1837. This consists of lithographed sheets given to the students of the École des Ponts. A few years previously Vicat had made his crushing attack upon the accepted mathematical—the Bernoulli-Eulerian—theory of beams. Here we find this attack justified and replied to by the introduction of the neglected slide (*glissement*) into the theory, and its application to a number of practical problems. Here, too, we see for the first time the true limit of elasticity expressed by a strain, and not a stress, maximum. This is a correction of the old theory which is of primary practical importance, although the old theory is still to be found in many English practical books, and even in such a theoretical authority as the German Clebsch.

A thorough appreciation of the true relation of theory to practice is evidenced by the following lines, which should be taken to heart by every technical teacher:—

"L'usage des mathématiques cessera de s'attirer des reproches si on le referme dans ses vrais limites. Le calcul pur est simplement un instrument logique tirant des conséquences rigoureuses de prémisses posées et souvent contestables. La mécanique y joint bien quelques principes physiques que l'expérience a mis hors de contestation, mais elle laisse aux expériences particulières le soin de déterminer quelles forces sont en jeu dans chaque cas, et il règne toujours à cet égard plus ou moins d'incertitude qui affecte nécessairement les résultats. Ces résultats ne doivent point être considérés comme les oracles, dictant infailliblement ce que l'on doit décider; ce sont de simples renseignements, comme les dépositions de témoins ou les rapports d'experts dans les affaires judiciaires, mais des renseignements extrêmement précieux et dont on ne doit jamais se priver, car il est extrêmement utile à la détermination que l'on a à prendre, de connaître la solution exacte d'un problème fort rapproché de celui qui est proposé, et de pouvoir se dire, par exemple, 'si les efforts étaient exactement tels ou tels, les dimensions à donner seraient telles ou telles.' De cette manière le champ de *l'appréciation instinctive* se trouvera réduit aux différences qui ne peuvent pas être le sujet du calcul théorique; et l'on voit que ces deux méthodes, loin de s'exclure, peuvent concourir ensemble, se suppléer et s'aider mutuellement, se contrôler même quelquefois,—enfin contracter sous les auspices du bon sens, une alliance féconde en résultats utiles sous le double rapport de la convenance et de l'économie."

These words represent exactly the spirit with which Saint-Venant entered upon the important investigations of later years. Of other earlier work of Saint-Venant, we may especially note the series of papers in the *Comptes rendus* for 1840–50. These contain the rectification of the theory of elastic rods by the introduction of the third moment in the case of inertial æolotropy in the section,[1] the complete equations for spiral springs, and the first rectification of the theory of torsion by the discovery of the distortion of the primitively plane section. These researches are all epoch-making in the theory of elasticity. To the next decade belong the classical memoirs on "Torsion" and "Flexure," the complete treatment of torsion on the basis of the distortion of the plane sections, and the complete treatment of flexure by the consideration of slide. The beautiful diagrams of the contour lines are known to all students of physics, if not from the original memoirs, at least from the "Treatise on Natural Philosophy" of Thomson and Tait. The very perfect plaster models prepared under the direction of Saint-Venant to illustrate flexure, torsion, and resilience, are less generally known,[2] but for teaching purposes they are of even greater value than the diagrams. In addition to these *opera maxima* we may mention the important researches on impact, belonging to the same period (Société Philomatique, 1853 and 1854). The decade received its final touch in the first volume of Saint-Venant's edition of Navier's "Leçons." This volume presents the first history of elasticity in the brief but luminous "Historique abrégé."

The last period of Saint-Venant's work contains the all-important memoirs on the distribution of elasticity in æolotropic bodies, on the various types of homogeneity, further researches on longitudinal impact, the tract on the undulatory theory of light, the second volume of Navier's "Leçons," the treatise on elasticity in Moigno's "Statique," and, amid a variety of *opuscula*, to crown the work of a life, the annotated edition of Clebsch's "Theorie der Elasticität." The original "Clebsch" contains 420 pages, the annotated translation with a much larger page has more than 900 pages. When will an English elastician arise, who will annotate Saint-Venant as Saint-Venant has annotated Clebsch?

One word more with regard to Saint-Venant's position as an elastician. In England the controversy over the number of elastic constants seems to have been decided in favour of multi-constancy. Saint-Venant was, and remained to his death, a supporter of the French, or rari-constant, hypothesis. The experiments, he argued, upon which the multi-constant elasticians based their theory were not made on truly elastic bodies, or were made upon bodies like wires and plates which are not isotropic. Into his treatment of such bodies he introduced, not the two constants of isotropy, but the constants of a cylindrical or planar distribution of homogeneity. Being written to last September on this point, he replied:—

"Vous voulez bien me demander si je conserve les mêmes opinions que j'ai exprimées et longuement developpées à l'appendix V. de mon édition de Navier, à savoir, la réduction des coefficients des formules d'élasticité à *un seul* (où $\lambda = \mu$ dans les formules de Lamé); ce qui conduit d'après le même principe que chaque action entre deux molecules est fonction de leur seule distance mutuelle, à réduire pour la contexture hétérotrope le plus générale, et à ne reconnaître que 15 coefficients distincts et non pas 21.

"Je reponds *oui* pour les vrais solides (supposés isotropes) comme sont les métaux ordinairement, ainsi que le marbre, le verre; mais *non* si l'on veut absolument par un motif quelconque que je ne conçois guère, appliquer les formules de l'élasticité au caoutchouc, aux gommes molles, aux gelées, et aux autres corps mous et élastiques, car ces corps-là ne sont que des mélanges de tissues cellulaires, de membranes élastiques, et de fluides visqueux que leurs cellules contiennent."

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[1] A bibliography of his memoirs relating to elasticity and the strength of materials will be given in the "History of Elasticity." A complete bibliography to 1864 will be found in "Notice sur les travaux . . . de M. de Saint-Venant," Paris, 1864. This is brought up to 1885 with partial completeness in the copy presented to us by Saint-Venant himself.

[1] He shares the honour of this discovery with Bellavitis.

[2] We have a copy of the whole collection at University College.

Perhaps the constant-controversy is not quite so obviously settled as some English physicists seem to think. But, however the future may regard it, history will record that on January 6 of this year died one of the greatest mathematical physicists and undoubtedly the greatest elastician that Europe has seen since the age of Poisson and Cauchy.

KARL PEARSON

ON SOME INTERESTING CASES OF MIGRATIONS OF MARINE FISHES, ON THE COAST OF VENEZUELA AT CARUPANO

CARÚPANO is a thriving seaport on the northern coast of Venezuela, midway between the peninsulas of Araya and Paria, in lat. 10° 14′ 15″ N., and long. 63° 18′ W. from Greenwich, and therefore in close vicinity to the channel which leads from the Atlantic into the Caribbean Sea between Tobago and Grenada on the one side, and Trinidad and the South American mainland on the other side. Through this channel enters the great western current of the Caribbean Sea, running at the rate of about one mile and a half an hour, though not with much regularity. The coast-line forms the western prolongation of the northern shore of Trinidad, trending almost due west. The water is rather shallow to a considerable distance from the land, the 100-fathom line due north of Carúpano being about 60 miles off the shore,[1] which gives for the sea-bottom a gradient of but 1·67 per 1000.[2]

Such are, in a few words, the general hydrographic conditions of that locality, famous in this country for the frequent occurrence of immense shoals of fish of different kinds, which, either alive or dead, are finally thrown by the surf on the beach. The inhabitants call the shoals *ribazones* when the fish arrive alive; in the other case they are called *turbios*, on account of the turbid appearance the sea presents in such circumstances.

Though the *ribazones* may occur at any time of the year, it appears that there is a greater probability of their advent during the rainy season, or from May to November. The weather must be fine, with a moderate breeze from the sea. The shoals are composed of a great many different species; most of the fish are, however, of small size. They are followed by large numbers of predatory denizens of the deep, sharks being generally prominent amongst them. In some cases the presence of whales has been recorded; it is undoubtedly the cachalot (*Catodon macrocephalus*), which occasionally visits the Caribbean Sea.[3] At the same time large flights of sea-gulls accompany the shoal, picking up a considerable number of fish, and, with their deafening shrieks and endless whirls, adding to the picturesque vividness of the scene. Owing to these manifold persecutions the frightened fish make towards the shallow water of the shore in such haste and with such impetuosity that the sea is almost boiling with foam for many miles. Most of the fish are still alive when they reach the beach, where the inhabitants, gathered in large crowds, are not slow in securing as many as they are able to carry away. By far the greatest number die on the shore, however, and their remains form a true *cordon littoral*, several feet in width and height, which soon fills the air with a most offensive smell.

The latest *ribazon* occurred on the morning of Oct. 10 last. The shoal seems to have come from the north-west, and was extraordinarily large. It contained such species as the "pargo" (*Lutjanus profundus*, Poey, the same as *Mesoprion aya*, Cuv. and Val.), which, as I am informed, had never been observed in any other *ribazon*. The Royal Mail-steamer *Severn*, going eastward at a speed of about eight miles an hour, was for two hours (from seven to nine in the morning) literally cutting through the shoal, and as she struck its main course at an angle of 45°, approximately, the breadth of the shoal must have been ten miles at least.[1] The shoal finally ran on shore to the east of Carúpano, and such was the quantity of stranded fish, especially between the places called Hernan Vazquez and Guayacan, that the local authorities deemed it necessary to summon a large number of workmen, in order to have trenches dug in which to bury the dead fish.

With respect to the causes of these migrations, I think they cannot differ from those which give rise to the well-known migrations of marine fish in other parts, the search for food being no doubt the most important. The waters of the great western Caribbean current are richer in food than the comparatively quiet part of the sea north of the current. The fish travel, therefore, in this direction, and attract of course a gradually increasing number of their enemies. On the coast of Carúpano the fisheries are insignificant, and thus the shoals are not disturbed in their migrations until they reach the shore. Farther to the west, in the waters of Margarita, the case is different, the large fishery establishments of this island having constantly boats and crews in readiness to intercept the shoals, as soon as their arrival is announced by the fire-and-smoke signals of the look-out men stationed on the different headlands and other places known to be favourably situated. With regard to Carúpano, it is certainly a great pity that the fisheries are so neglected that every year a large amount of what ought to be a rich harvest is lost, and left to turn into noxious and fever-breeding carrion.

The *turbios* are *ribazones* during the arrival of which submarine eruptions of deleterious gases, principally sulphuretted hydrogen, happen to break out, so that the fish are killed before they reach the shore, the water at the same time becoming turbid with the mud from the bottom, which is violently stirred up by the outbreak of the gas. The quantity of the latter must be very considerable indeed, as the foul smell on such occasions is noticed all over land and sea.

Gaseous eruptions of the same nature, as well as sulphurous springs, are not at all uncommon in the neighbourhood of Carúpano; the two *azufrales*, about twenty miles to the south of the town, being the most important.[2] The whole region, in fact, was at the end of the Tertiary period the theatre of a great geological catastrophe, when the Golfo Triste was formed by a sudden subsidence, which was also the origin of the so-called delta of the Orinoco,[3] the Gulf of Cariaco, and the many lagoons in the provinces of Cumaná and Maturin. At the same time extensive tracts of land were submerged to the north of the mountains which run through the whole length of the peninsulas of Araya

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[1] "Deep-Sea Soundings in the Gulf of Mexico and Caribbean Sea" (Chart No. 21 in *Report* of the U.S. Coast and Geodetic Survey, 1881, Washington, 1883).

[2] I take the nautical mile, equal to one-sixtieth part of the length of a degree on the great circle of a sphere whose surface is equal to the surface of the earth = 1853·248m. or 6080 70 feet (J. E. Hilgard, "On the Length of the Nautical Mile," in the *Report* mentioned in the foregoing note, pp. 354-356).

[3] There is in the Museo Nacional of Caracas a tooth of this species, from a specimen which was stranded some years ago on the shores of Margarita. I saw myself, in 1873, another whale, but only for about one minute, in the sea between the islands of Tortuga and Margarita. The animal swam with its mouth wide open and the head partially raised above the surface of the water. The upper jaw was small and narrow, so that it certainly was not a cachalot. Though I could not see the back, I supposed then it might be a Balænoptera, perhaps *B. rostrata*. However, I do not know whether this species comes so far to the south.

[1] From information given to me by Señor Bastardo, a medical student at our University, who was a passenger on board the *Severn*.

[2] Wall and Sawkins, "Report on the Geology of Trinidad," London, 1860, p. 198.

[3] The Orinoco has no delta in the true geological sense of this word, as the land comprised between the outer branches of the intricate fluviatile plexus of its mouth has *not* been formed by the river. The southern branch is the old river channel; when the above-mentioned subsidence took place, the land on the left bank sank gradually towards the north, and part of the waters, following the new slope of this northern plane, cut into it the different channels with their connecting branches which, after a slow and tortuous course, empty into the sea between the old mouth of the river and the southern entrance of the Golfo Triste.